

Gutberlet's principle

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The concept of infinity was always eluding satisfactory explanation, and it was always puzzling because of the paradoxes. At the same time, it was necessary to use it, especially in theological matters. Most efforts were directed at basing this concept on that of the finite, but none of them were successful. However, even if the concept of infinity was assumed as valid, usual attempts to harness its power were done by reducing it to potential infinity and then building the concept of actual infinity. Probably the first attempt to reverse the importance of these two views of infinity was undertaken by Konstantin Gutberlet.

Gutberlet (1837–1928), who was a Catholic priest and for many years (1862–1924) a professor of theology, philosophy, and dogmatics in a seminary in Fulda and Würzburg, was interested in the problem of infinity as a theologian and a dogmatist. As he wrote, „all my studies and works have a clear goal: to defend the Christian world view, not to solve purely scientific problems“¹. He was a prolific writer and translator, but he did not consider himself a revolutionary in theology and philosophy; he wanted to keep his views close to *sententia communis*, but he saw three areas in which he broke away from this *sententia*. One of these areas is the relation between potential infinity and actual infinity.² Because of Gutberlet's originality in this area, his statement is termed here, Gutberlet's principle.

1.

What is called here Gutberlet's principle is a very forceful statement concerning the priority of actual infinity over potential infinity. According to Gutberlet, the statement about a set being infinite only potentially is contradictory, and one should rather say that „a magnitude can be called potentially infinite when it has a foundation in corresponding actual infinity. Otherwise, why is it possible to always surpass *every* limit which is set in an infinite extension? Because behind any growing extension there is always some more extension“ (U 11; M 219–220).³ However, in surpassing any limit, one does not create a new extension or a new, larger and larger set, but only recognizes (anerkennt) that which already exists, that which is „objectively possible.“ Limits that can be potentially overcome can be in reality overcome if there is a background into which these limits can be moved. Otherwise, the reality of the area which supposedly exists *in potentia* would have to be created by the power of our minds.

As a Thomist, Gutberlet maintains that this very strong statement would be acceptable to Thomas Aquinas. Thomas says that if a body moves by one palm, it gradually leaves the area of the first palm to move into the area of the second palm. However, every point of the first palm is the beginning of a place. „Accordingly, since magnitude is infinitely divisible, and

¹ Konstantin Gutberlet, *Eine Selbstbiographie* (Fulda 1930) (published posthumously by Karl A. Leimbach) 6.

² *Ib.*, 177; the other two areas are the necessity of revelation to the act of salvation and the problem of mortality of animal souls.

³ Hereafter, the following abbreviations will be used:

M – Constantin Gutberlet, *Allgemeine Metaphysik* (Münster 1906 [1879])

P – Constantin Gutberlet, *Die Psychologie* (Münster 1890 [1880])

ST – Thomas Aquinas, *Summa theologiae*;

U – [Constantin] Gutberlet, *Das unendliche, metaphysisch und mathematisch betrachtet* (Mainz 1878).

the points in every magnitude are likewise infinite in potency, it follows that between every two places there are infinite intermediate places“ (ST 1.53.2). This may be strengthened by saying that because the example concerns evidence „from sensible experience,“ the intermediate places or points have to be there in order for the body to be able to change places and move from one area to another; that is, the infinite places are actually there, and hence actual infinity would be the ground for claiming that the infinity of points is there in potency.⁴

There is a problem, however, in defining an ontological status of the actual infinity. In a word, actual infinity is something real, but not existing. „Between pure nothing and existence there is an unlimited area of the possible with its intelligible, metaphysical being that is independent of our thinking as much as, or even more than, the world of what exists“ (U 15). In particular, extension is not nothing, otherwise how could physical bodies be located in a nothing?

Gutberlet's solution is dangerously close to wordplay since it may be interpreted as a statement that actual infinity is the infinity of the possible. Why, we may ask, not ascribe potential infinity to the „unlimited area of the possible“? Joseph Bloch – who treats Gutberlet's views not just critically, but very contemptuously – claims that there is nothingness between pure nothing and the existence, and an attempt to insert the possible between the two is an attempt to have „an observer standing outside any borders, who trembles between being and nonbeing.“⁵ Gutberlet's struggles come from the fact that he wants absolute infinity to have an *objective* status, independent of our thoughts, so he makes the possible real. „The possibility of things is not pure nothing and not an abstract idea, but something very real, an objective given, but [at the same time] not something existing“ (M 275). The potentiality is a possibility of existence, but this possibility has to be backed up by something real, otherwise how could it be actualized? We say that the falling of a ball onto the ground is possible since we know the workings of the real law of gravity that causes a change in the position of the ball when it is released. It is possible for an aircraft to fly since the real laws of aerodynamics working in the real world make such a possibility become reality, so in this way, even before the time of Icarus, an aircraft was not nothingness, but a reality ready to be actualized. This aircraft was, so to speak, inscribed into the laws of aerodynamics as their yet to be actualized consequence. On the other hand, creating a circular square is impossible because it would defy the laws of geometry. Some possibilities, to be sure, may not be recognized at a certain time – the progress of science and technology consists in determining that some possibilities are real, not figments of our imagination, and that they are allowed to become a reality through the nature of the laws of nature. The potentiality would thus resemble unfolding a scroll and seeing its content – but the scroll has to be there to start with. Similarly, any „possibility of things“ is possible if an existing reality stands behind them. A physicist would say that this reality is a reality of nature and its laws, while a theologian would say that it is reality of God and his nature, a creator of nature. The potentiality has, therefore, a borrowed reality; the potentiality by itself would be incapable of bootstrapping itself to the level of actualization; it could not bring into existence what it carries in its womb. In this sense, as it appears, we can understand Gutberlet when he writes that „when actuality of the possibility already on hand should *all by itself* produce a new actuality, then – because each thing acts inasmuch as it is actual – it must be at least as actual as the ac-

⁴ Cf. C[aspar] Isenkrahe, *Untersuchungen über das Endliche und Unendliche, mit Ausblicken auf die philosophische Apologetik* (Bonn 1920) v. 2, 146.

⁵ Joseph Bloch, *Die Entwicklung des Unendlichkeitsbegriffs von Kant bis Cohen* (Berlin 1907) 40; through Gutberlet's solution, he says, „devil is cast out with Beelzebub“ (39). Cf. unsympathetic remarks concerning such a solution in A[drian] W. Moore, *The infinite* (London 1990) 208.

tuality that is to become, that is, there does not emerge any actuality [just] through a *change*, but *there is* actuality in what is already there, [actuality of] what it could become“ (U 181). Thus, in the case of potential infinity, there should be an actual infinity already in the background. This infinity, however, is not to be found in nature, and Gutberlet many times vehemently opposes such a possibility. Therefore, it is inescapable for him to posit the existence of God and make him a guarantor of reality in general and of reality of actual infinity in particular. The reality of God guarantees the reality of the possible in some not very clearly defined way (can possibilities be considered God's ideas after it was stated that they are not abstract ideas?). That is, in effect, what Gutberlet will do later.⁶

Gutberlet emphasizes that the actual infinity in the physical world is not critical to prove his point. He uses extension for the sake of example, but numbers could serve his purpose just as well and „certainly nobody will deny [the existence of] possible and yet true numbers“ (U 16). In the realm of existence only God is infinite; all creation is, however, finite (U 24). Potential infinity does not express the order of things, but rather the relation of minds to things; „the essence of potential infinity amounts to the successive and endlessly expandable limits (*fortsetzbaren Fixierungen*) that the mind sets in the actual infinity“ (U 28). Therefore, the mind could not have divided a continuous line indefinitely if „an objective possibility“ had not been already there (U 31).

If the reality of infinity cannot be found in the material world, it should be looked for in the transcendental world, in what Gutberlet calls the metaphysical area. „Metaphysical area with its beings and principles ... is not a mere thought norm ... but it has a being independent of thinking and existence“ (U 191). This being independent of thinking and existence exists from eternity, that is, objectively. This being is infinite extensively, because the set of possible things is infinite, and intensively, since every level of perfection can have a level surpassing it, at least as a possibility. „Therefore, there must exist an absolutely infinite being. For a possible being cannot by itself exercise an irresistible power on thinking and events in the existing world that we experience and observe“ (U 192). This all-powerful being sets all norms of thinking and laws of being. Therefore, every being (possible and existing) is created in likeness of this ideal being and all events in the world have their model in it. Thus, the ideal being is a counterpart of Plato's world of ideas, and it is not surprising that Gutberlet calls him divine although Plato exaggerated with his „flight of an eagle toward blinding sun“ (U 191).

This ideal being, however, appears to be the means through which a real being influences reality. Such a conclusion is inescapable because the ideal world appears to be, to Gutberlet, the world of the possible, being real, although not existing. As such, it has no power by itself to actualize anything, and hence a real being makes use out of it to extend and modify the extent of existence. Importantly, because the ideal world is infinite, so must the real being be infinite. Secondly, because the ideal world is ideal, the real being must exceed its perfection.

Gutberlet's goal is to firmly ground the reality of the ideal world that occupies the position between nothingness and existence, which includes what is possible and thereby real. However, he often uses our thoughts as an argument to substantiate the reality of this world. This world should be independent of our thoughts and yet our thinking seems to be the strongest argument in support of its reality. For example, Gutberlet writes that „this

⁶ It is worth noting that Aristotle struggled somewhat similarly with the concept of motion. Motion was to him the actualization of the possible in so far as it exists potentially. But in discussing the problems with defining this concept, he notices that it is because motion can be placed neither among potential beings nor actual beings. There would be a third category then, if only for motion itself, and that is the reason why this is „hard to discover, but what can exist“ (*Phys.* 201a-b; *Met.* 1065b-1066a).

being is generally what everyone thinks when he thinks about being, in particular it is *being human*, being substance etc., namely being which presents itself to us clearly when we think about an ideal being“ (U 191). The thrust of the argument seems to be that the ideal world enables our thinking, although this thinking is the prime witness of this world. We can think about something *because* through our thinking we refer to the ideal world. It is like unfolding a formal theory defined in terms of a set of axioms and rules of inference. Potentially, in an ideal form, all the consequences are already contained in the axioms, they are inferrable from the axioms through the application of the inference rules; they are contained in the axioms in a dormant state from which our thinking awakes them. It is not unlike Popper's third world from which human ideas stem. In this way, an intuitionist can intuit his intuitions if there is an objective possibility for such intuitions to be made. A constructivist can construct his constructs if there is a non-subjective ground for molding such constructs. A constructivist cannot construct in mathematical vacuum attempting to bring to being any theoretical construct at will. Theoretical freedom is here constrained by the nature of mathematical objects and, in mathematics, freedom is attained not by ignoring any constraints but by making them work in accordance to the mathematician's goals.⁷

Gutberlet uses here a theologically potent argument. If an underlying actual infinity were not assumed in the process of going beyond every limit of a growing extension, then either the mind would have to make what it accepts, or – which amounts to the same – it would have to assume the existence of a reality where there is none. Hence, the mind is never truly creative; it cannot surpass limits if there is nothing already there. The human mind cannot create anything truly new; otherwise – a thought unacceptable to a theologian it would do something which God did not do. God is the only creator, and man at best can discover and acknowledge what already has been brought into being. Hence, potential infinity is merely an unfolding of actual infinity; potential infinity is a way in which actual infinity manifests itself to the finite mind of men. In this sense, „what is possible, is necessary,“ a thought which was used by Leibniz in his proof of God's existence.

And still, Gutberlet's reasoning seems to rely too much on the subjectivity of thinking to prove objectivity of its content. He says that „we can always think about more and more perfect beings, without end. Therefore, there are *possible* beings that are infinite in their magnitude and perfection“ (U 194). These possible beings are real beings, independent of our thought; but are they real *because* we think about them? The argument is similar to Anselm's ontological proof: because we can think about a perfect being, therefore a perfect being exists since we are imperfect; otherwise, how could imperfection breed perfection? Gutberlet adds here a new twist: since we can think about infinity, then an infinite metaphysical realm of the possible must exist – and therefore an absolutely infinite, existing Creator of this realm also must exist – because we are finite beings; otherwise, how can the infinite be bred by the finite? If he repeated after Descartes that infinity means God's perfection, then the proof would be almost exactly as Anselm's: if we think about infinity, that is, perfection, then we must think about God's perfection, that is, about God himself. To Descartes, there was nothing clearer than infinity (a clear idea, although incomprehensible), to the extent that infinity allows man to understand the finite, which elsewhere is termed Descartes' principle.⁸

Gutberlet, without referring to him, readily accepts Descartes' principle when he writes that „no speculation ... can rest on the finite as the last ground of things. Everyone expli-

⁷ See also Adam Drozdek, Thomas Keagy, A case for realism in mathematics, in: *The Monist* 77 (1994) 329–344.

⁸ Adam Drozdek, Descartes: mathematics and sacredness of infinity, in: *Laval théologique et philosophique* 52 (1996) 167–168.

citly or implicitly takes infinity to be the end of being and thinking. The difference of world views stems only from different framing of infinity itself and the transfer of infinity to various subjects that results from this [framing]" (U 212). Moreover, „every case in favor of the existence of a finite entity is also a case in favor of infinity since the latter includes in a virtual manner all perfection of all finite entities and thus every time a finite entity exists, so must also exist an infinite entity; therefore, the finite is impossible without the infinite" (U 162). The number of possibilities cannot be limited to any amount of finite entities because for any limitation one can envision a new possible entity made out of possible copies of possibly existing finite entities, whereby the number of possible finite entities can be indefinitely extended. Therefore, the potential infinity of finite entities leads us directly to an admission of actual infinity: an actual infinity of the number of possibilities. Thus, unless we restrict the constructive powers in the realm of the possible – and we need only produce possible copies of possibly existing entities and then put these copies together to create new entities, and all this, again, in the realm of the possible – we are forced to accept that the actual infinity precedes the potential infinity.

2.

Mathematics postulates actual infinity (U 53), claims Gutberlet, and this claim is not unjustified since working with the assumption of actual infinity is common place in mathematics. Dedekind's definition of real numbers (the now so called Dedekind's cut) relies on the assumption of actual infinity of rational numbers. Although Weierstrass defines real numbers differently, he also uses actual infinity by defining irrational numbers as infinite groups of rational numbers.⁹ Another example in mathematics of using the postulate of actual infinity, the example analyzed at length by Gutberlet, is the theory of infinite series. For instance, if one says that $\pi/4 = 1 - 1/3 + 1/5 - 1/7 + 1/9 + \dots$, then both sides are truly equal if the series included all the terms (U 62). Also, set theory is based entirely on this assumption and Cantor was a great supporter of Gutberlet's principle.¹⁰

Although an actual infinity exists, according to Gutberlet, this privilege is not shared by infinite numbers and therefore „it is very incorrect to talk about an infinite number" (U 18), which is a view similar to Leibniz'. The reason is that the number is a „defined, specified set of units" (U 18). In denying the possibility of an infinite number, Gutberlet is in agreement with Thomas, whom he quotes as saying that „no species of number is infinite" (ST 1.7.4). He is, however, in conflict with Thomas, who states – which Gutberlet does not quote – in the same article that „it is impossible for there to be an actually infinite multitude, either absolute or accidental." Gutberlet tries to overcome this conflict, which he clearly realizes, by using Thomas' criterion of existence, which is definability: everything must be of a particular kind or species. An infinite set is thereby defined as that which is „above all species of finite numbers, therefore, defining it through a particular species of number is not only superfluous but nonsensical" (U 32). This may be considered an eyebrow-raising solution to

⁹ Carl B. Boyer, *The history of the calculus and its conceptual development* (New York 1959) 290; Morris Kline, *Mathematical thought from ancient to modern times* (New York 1990) 983, 987; Michael Hallett, *Cantor's set theory and limitation of size* (Oxford 1984) 29–30. Also Gottlob Frege supported the assumption of actual infinity, *Kleine Schriften* (Hildesheim 1967) 163 and *Posthumous writings* (Oxford 1979) 68–69.

¹⁰ This principle is called the domain principle by Michael Hallett (op. cit., 7), who sees in it one of the basic principles of Cantor's philosophy of mathematics, but he does not recognize that the principle originated with Gutberlet, although he mentions him twice in his book.

overcome Thomas' limitation by Thomas' criterion: what is undefined is defined by the fact that it is beyond the scope of the defined. This allows Gutberlet to say that even God cannot assign any number to the set of all possible things, but the set itself is determined by that „it [or, rather, its cardinality] is above all numbers“ (M 221). On the other hand, such an approach may be defended on the ground that, after all, *affirmatio est negatio*. Definition is also always a criterion – although not always easily applicable – of what is of certain kind, thereby also defining what is outside of the kind. If we define a circle then, by the same token, we also define what is not a circle.

The reasons for not ascribing numbers to infinite sets are concerned with paradoxical results, some of them known for centuries. For instance, an infinite number could be broken down into a sum of odd numbers or a sum of even numbers (U 34). An infinite number can be an infinite sum of ones or an infinite sum of tens (U 36). An infinite number can be an infinite sum of a finite number or an infinite product of the same number (U 42). It did not occur to Gutberlet, as to most of his predecessors, that arithmetic of infinite numbers can be paradoxical, after all, without being contradictory. Gutberlet himself uses an equation $1 + 2 + 3 + \dots = \infty$, but he hastens to add that „it cannot be stated with certainty *how large* this sum is“ (U 87). Thus, either the sum is not a sum, and an infinite number of summations – although real because actual infinity is presupposed – is impossible, or the equality is only metaphorical because a sum of numbers would amount to a nonnumber ∞ .

Gutberlet's solution is unacceptable to Cantor who says that „no one can do the opponent of the transfinite greater favor as by this solution, since the set and the number are indissolubly interconnected; if we renounce one then we have no rights to the other.“¹¹ If a set has elements then it should be possible to express their number, even if the set is infinite. The source of the problem was diagnosed by Cantor, who calls it a prime falsehood. The falsehood consists in „ascribing in advance properties of finite magnitudes to actually infinite magnitudes, from which a contradiction with its non-finitude is easily deduced.“¹² The same falsehood is used in proofs against actual infinity in nature. In this category Cantor includes Gutberlet's infinite wire argument.¹³ If such a wire (line) existed, says Gutberlet, we could cut out its piece and tie together the separated segments, whereby they would be moved toward one another to make it possible. But now these segments are not infinite anymore, since a part of their infinity was removed, „both are missing so much of infinity as they were moved toward the middle. Therefore both are limited on the side of infinity and also in the middle,“ thus, they are finite. But, responds Cantor, the argument holds only for finite lines. If a line AO stretching infinitely to the right is moved by an interval to the left, then, true, each point on the line is moved to the left by this interval, but this is not true about point O infinitely removed from A; the point O remains in its place, in spite of the movement of the line.¹⁴ If the line has its counterpart in the transfinite number ω , then the entire reasoning means that $1 + \omega = \omega$, which is true in set theory. Notwithstanding Cantor's argument, Gutberlet writes in his apology textbook that „it is absolutely impossible that the number of celestial bodies of atoms is infinite or that the existing matter has at all infinite extension,“ which he shows using Cantor's version of the wire argument:

¹¹ Georg Cantor, *Mitteilungen zur Lehre vom Transfiniten* (1887), in his *Gesammelte Abhandlungen* (Hildesheim 1962) 394.

¹² *Ib.*, 396.

¹³ Hugo Bergmann writes in his book *Das Unendliche und die Zahl* (Halle 1913) 51 that „nothing is clearer than [the fact] that Gutberlet treats here infinity as a finite magnitude.“

¹⁴ The argument from a point at infinity was not convincing to Cantor himself, who later never used it, cf. Robin Small, *Cantor and scholastics*, in: *American Catholic Philosophical Quarterly* 66 (1992) 427.

if one moved an infinitely extending existing line by 10 units, then the line would not extend into infinity any more.¹⁵ Gutberlet's argument may, however, be reversed: when an infinite line becomes finite after moving it by 10 units, can it be turned into infinite by moving it back by the same number of units? If the hypothetical assumption of the line being initially infinite is to stand, then it should have – by definition – no limits.¹⁶ Do these limits appear after moving the line? Can the limits be removed after moving it in an opposite direction? Pulling an infinite line by 10 units does not pull the end of the line closer to us because the line does not have any end. Therefore, it appears that Gutberlet does not treat seriously, or literally, enough his initial assumption about the line being infinite, otherwise changing lines from finite to infinite would be all too easy, even without the need of assuming actual infinity at the outset. And this would thwart the validity of Gutberlet's principle.¹⁷

3.

Potential infinity presupposes actual infinity, but the latter cannot be found in the material world. The only truly infinite being, that is not only infinite but also existing, is God himself. His infinity manifests itself in our awareness of potential infinity and in assuming it in science, in particular, in mathematics, but infinity proper is, quite literally, out of this world. Many of Gutberlet's deliberations concern proving this point. To him, as a theologian, this is the centerpiece of his analyses: showing that God by his nature is infinite. However, his analyses are not always convincing.

A being existing through itself is the being that has a ground of its existence in itself and that exists by necessity (U 133). Matter was created out of nothing, therefore, it was created by a spirit (U 137). Commenting on the words of Thomas, „to create can be the proper action of God alone“ (ST 1.45.5), Gutberlet says that the act of creation is universal; universality of effects requires appropriate universality of power. A cause of all things and of all kinds must be thus an infinite being (U 139). However, it does not seem that „with the universality of power goes perfection at the same pace. No finite power is so perfect that there could not be a greater power; therefore, power of creation is infinite.“ This does not follow. If the world is assumed to be finite, then the creating power does not have to be infinite to create it; to use today's terminology, the creating power could transform some of its energy into matter, which does not require infinity of this power. Maybe the theodicy in the spirit of Leibniz could do: it could be claimed that our world is the best of all possible worlds, where the number of possibilities is unlimited, that is, infinite, and that a power has to be able to compare all these worlds to choose the best, as in the metaphor of a pyramid of possible worlds described by Leibniz in concluding paragraphs of his *Theodicy*. It appears that Gutberlet is close to this mode of thinking, after all. He emphasizes that the weight of proof lies in universality of objects, „that is, in freeing from limitations to particular kinds of things“ (U 141). That seems to mean that God not only created what actually exists but what could exist; after all, existence „in reality extends to *all possible* beings“ (U 140).¹⁸ Furthermore,

¹⁵ C. Gutberlet, *Lehrbuch der Apologetik* (Münster 1888) 147–148.

¹⁶ Infinity is defined as being without end, without limits (U 10, 16; M 216–218).

¹⁷ Cf. Isenkrahe, *op. cit.* 186.

¹⁸ Gutberlet is concerned about the fact that Leibniz' optimism leads to fatalism, thus, although he is willing to accept Leibniz' optimism, Gutberlet stresses God's freedom in creating the world by stating that God had to „choose freely among all possible worlds the one of a specific, finite perfection, and this freely chosen [world] can, of course, in many respects be called the best.“ Constantin Gutberlet, *Die Theodicee* (Münster 1909 [1878]) 310; cf. also ST 1.25.6.

because the number of such possibilities is infinite, so is the creating power. This, however, may not be a convincing argument if the role of the possibilities is not as clear as in Leibniz. What are these possibilities for? For Leibniz, they are used to choose the best world that can be. For Gutberlet, it seems, they are possibilities to show that infinity is – or can be – created, thus their role is to show the infinity of the Creator. However, nothing prevents us from a conjecture that the number of possibilities is finite (it can even be done in Leibniz' system, thereby somewhat undermining it); in this way they could not be used in proving God's infinity.

A more interesting argument for God's infinity is the use of the difference between the possible and the actual. Gutberlet says that bringing into being a possibility requires a power proportional to the distance between the possible and the real. This is true if possibility means a more or less difficult task, such as building a house vs. building a city. In this case the proportion holds. However, this is somewhat misleading since Gutberlet means to put all the possibilities on a par, possibility being something not yet existing, „nothingness as possibility,“ in which case the distance between the possibility and the real, says Gutberlet, is infinite (U 147). In support of his argument he refers to the concept of kinetic energy and that of power (although he confounds the two, U 153). Kinetic energy $K = mv^2/2 = \text{mass} \times \text{velocity}^2/2$, work $W = K - K_0$ and power $P = W/t$. The last formula indicates that the shorter the time is, the greater power is needed to perform work W . In the act of creation, „now there is nothing and immediately after it (darauf) the creation is there in its entirety,“ which means that because this act was performed in literally no time ($t = 0$), then the energy must be infinite (U 153). The argument is interesting, but it must be treated with caution, at best as a metaphor. The creation was brought into being along with time, therefore, time-related laws can only be applied if time exists, otherwise, the variable t is void rather than zero. The same goes for all physical laws, for that matter; they are applicable in nature, thus when nature is not there, the laws are not there either, or at least, their validity is suspended. Their validity is not general, by the way, even in nature itself, to mention only singularities, such as black holes. The moment of creation is a singularity of singularities, therefore, using the laws of physics to prove a point concerning the nature of this singularity has at best a figurative meaning. Therefore, the use of a formula for power is not conclusive in a literal sense since when assuming that time is zero, not a limit, then the formula does not prevent us from stating that the power is finite; dividing work W by zero gives an undetermined result, thus W could have a finite value just as well.

„The existence of a being, a ground of all beings, cannot be limited to the perfection of any of the kinds of what exists, but it must include them all in some way“ (U 164). In the sequence of kinds ordered by increasing perfection, each of the possible kinds could be claimed to be the one in which a being existing by itself appears; therefore, this being must be above all kinds, thereby being simply being. This being is not above all kinds in the sense of being a transcendental concept common to all entities of all kinds since it would be tantamount to pantheism and it could not be a cause of the world. It encompasses perfection of all possible perfections, it is being without additional qualifications. Such a being must be infinite.

Following Joseph Kleutgen who, in turn, follows Plato and Thomas, Gutberlet shows that God is existence itself and thereby he is infinite. „What can be pronounced about many things cannot make any one of the many to be what it is“ (U 165), that is, what can be predicated about many things, can be caused in all things and thus does not have to be ascribed to them on account of their essence; there are needed some individualizing traits which distinguish one member of the many from the rest. Existence can be pronounced about many entities, even about everything, but existence can be pronounced only about One as its cha-

racteristic trait, as what makes it an individual being; in everything else existence is caused by this one being, it is infused into them from the outside. Existence is the self and the essence of this one being. „Such [a being] must therefore exist otherwise it could not be a cause“ of everything else (U 165). This being has the whole fullness of existence, that is, this being „is existence itself, an infinite and perfect being“ (U 166).

Another argument: If general concepts represent pure (lautere) perfections, like wisdom and power, then there can be a being, whose self is wisdom and power, who is wisdom and power. „But there must exist such a being; then actually existing wisdom and power must eventually be explained by a cause existing by itself, [that is, by] God“ (U 166). This being possesses these properties in their fullness, in an infinite degree.

4.

What is an origin of the idea of infinity? How do we come to the knowledge of infinity? Our reason by itself is *tabula rasa* (U 197). Gutberlet rejects the idea that any idea can be in-born, including the idea of God, and all ideas can be explained as derived from perception and working in connection with an „immaterial principle in man“ (P 134–135).

Gutberlet refers here to Aristotle who introduces the notion of νοῦς ποιητικός, *intellectus agens*, active reason whose task is to abstract *species*, or a concept, from *phantasmata*, to later turn toward these phantasms, or perceptual data, to illuminate them. Through this activity there emerges what the scholastics called *species intelligibilis* which is *expressa*, when it leads to cognition, that is, when the mind is determined by it to have an expression of the object; *species intelligibilis* is called *impressa*, when it is a habitual state in mind, or rather a habitual cognitive form in the mind that makes it possible for the mind to build again an actual representation according to an abstraction that once took place (P 150–151, 155). This *species intelligibilis* can also be considered the disposition of reason through which reason can become similar to an object and be able to reflect it; such an ability is necessary because reason must be similar to an object in order to know it.

Reason is an ability to recognize essences of things behind individual appearances. Our immaterial reason is a contingent, finite, and individual principle; that it can grasp (erfaßt) the infinite – and also the general and the necessary – we can recognize through its essence; and although this grasping is inadequate, it is nevertheless satisfactory due to reason's actual ability (P 156). This explanation is at best cryptic and Gutberlet nowhere explains how the concept of infinity may emerge in finite being, using finite powers of reason. He writes about abstraction as the mechanism of obtaining concepts, *species intelligibilis*. The abstraction is no cognition; it is rather a necessary and unconscious activity of mind through which cognition comes into being (P 156). The reason has an ability to abstract the general essence of material things, thus the general entities of reason exist in individual things. The content of the mind comes from senses and *intellectus agens* (P 157). There are two kinds of abstraction: natural, unconscious abstraction whose generality is not recognized (*universale directum*), through which we get our first concepts; and conscious, voluntary abstraction that uses comparison of essences to create more and more general concepts (*universale reflexum*) (P 159). Presumably, this is the abstraction which allows us to have an idea of infinity. If it does not accomplish it by itself, this type of abstraction is amplified by the operation of negation. Gutberlet says that first we recognize the positive in finite things, then „the negation of further reality in them, and thereby the finite. With this, as we think about the finite, the infinite as its negation forces itself (aufdrängt) upon us“ (M 216). How exactly the idea of infinity forces itself upon us remains unexplained.

How clear can the idea of infinity be? We are, after all, finite beings, thus, if infinity emerges by extension of the finite – as, for example, Gassendi and Locke proposed – then the possibility must exist that such an extension can be infinitely carried out. But a finite being cannot make such an assumption. Therefore, relying on extending the finite to the infinite, or abstracting the infinite from the finite, can never be actualized by finite beings. Therefore, it is difficult to see how in this framework an idea of infinity can be formed.

Knowledge can be acquired not only through the use of cognitive apparatus, but it also can stem from revelation. However, in the case of infinity, even revelation is not sufficient, and we should be distrustful of a revelation „in which the infinite would be presented as fully comprehensible (begreiflich),“ because „no finite being can penetrate (durchdringen) the infinite,“ which means that the concept of infinity is outside the limits of evidence.¹⁹

It appears that either our understanding of infinity is inherently dim and metaphoric or the concept of infinity must come from outside of cognition – be it inborn or from revelation. After all, Gutberlet is not against inborn traits in human beings. The entire cognitive apparatus is inborn. Moreover, there are some inborn drives which, as a matter of fact, define our humanness. „There are two drives implanted by God in the human soul, which unequivocally prove that he preserves for it another life kept for it.“ One is an „irresistible, necessary by nature, essential desire of happiness“; besides, there is a natural tendency, a drive for morality that „allpowerfully obligates us to exercise morality and virtue. These two natural drives (tendencies) are not thwarted and general experience shows that nature does not do anything in vain. Even to a lesser extent should they, as drives implanted in the spirit by God, be unneeded; and still to a lesser degree should they contradict each other because in both of them the infinite wisdom would contradict itself. Thus, if there is no other life, so would all these natural drives be thwarted and they would be in an unsolvable contradiction with each other“ (P 322).

Gutberlet states very strongly that the highest wisdom would contradict itself „in the most stupid manner“ if it gave people drives to happiness when reaching happiness would not have been possible. Our happiness is our driving force and experience along with „universal cognition power of our spirit“ is necessarily directed toward an infinite good. This „striving for God“ manifesting itself with „exception-free generality“ is implanted by God; it is a natural drive, as no other. Because this drive is of divine origin, it cannot be void, thus there must exist an infinite good which is an object of our happiness (U 170). The argument is not unlike Descartes', who stated that God does not deceive us in implanting in us goals which are unattainable. Gutberlet is, however, concerned with God's infinity, and his argument purportedly proves it by pointing to the infinity of the good which our happiness requires and, which Gutberlet does not state explicitly, that God himself is this good, thereby being infinite. This proof, however, is not independent of what he already stated. God is infinite wisdom, he is wisdom, and thus is infinite, therefore, such wisdom simply could not create in men natural desires which would be unreachable. The infinity of God's wisdom implies reality and infinity of goodness for which humans strive.

Moral feelings are also a witness of God's infinity. Moral convictions are associated with moral feelings of „unexplainable shame.“ This is conscience at work, a faculty that „possesses clear knowledge of God and a developed moral feeling.“ Moral precepts have an inescapable power over us witnessing an existence of an „absolute, almighty will“ which would be impossible without an infinite spirit. This is basically a Kantian argument: Kant was forced to make God, an infinite God, a postulate of practical reason to infuse universal validity into morality. Gutberlet, however, assumed already that God exists and attempts to prove his in-

¹⁹ C. Gutberlet, *Logik und Erkenntnistheorie* (Münster 1909 [1892]) 318–319.

finité. Kant postulated God, assumed his existence as a matter of theoretical necessity in the realm of practical reason and was only marginally interested in his infinity.

These drives can be truly human if they are God-directed, if their goal is to reach beyond the finite. „Happiness that we essentially and with necessity desire, is a perfect fulfillment of our highest abilities – of reason and of will – that because of their generality and infinity can find rest only in perfect happiness“ (P 326). Thus our highest abilities are infinite in that they are not limited by any bounds, by any worldly bounds, that is. Infinity of reason and will is rooted in the only actual infinity, the infinity of God. Thus infinity, although not as a concept, is built in to reason and will as potential infinity anchored in the actual infinity of God. Reason can recognize infinity, dimly and inadequately, but satisfactorily, because it itself is infinite. This dormant infinity of reason and will is thus an indispensable element of human condition because without it humans would have their eyes fixed only to the material side of reality. It goes almost without saying that the existence of an infinite good has practical consequences: if all earthly goods are of lesser quality, they should be abandoned in favor of the infinite good (U 172).

5.

At every step of his deliberations Gutberlet is a theologian interested in proving, or at least explicating, Christian truths. One of these truths is God's infinity. For Gutberlet, every thought about infinity is by necessity saturated with theological content. Even if a mathematician talks about infinity, he does not make innocuous, scientifically neutral statements, but, even if unawares, he makes theologically-laden presuppositions. Infinity, may it be potential infinity, assumes reality of actual infinity. Actual infinity may not exist, but it is still real, and this reality is guaranteed by a Creator, who is infinite and existing. There is, thus, a very short distance from mathematics, and also from sciences, to theology. That is what was very appealing to Georg Cantor, who saw in Gutberlet a theological ally. Cantor considered his set theory to be a powerful tool to be used in support of Christian faith,²⁰ and in Gutberlet's work on infinity he saw a strong support of his views. He wholeheartedly accepted Gutberlet's principle, and almost certainly under his influence Cantor himself wrote in his *Briefbuch* (1886): „In order for a changing quantity [the potential infinity] to be usable in any mathematical analysis, there must, strictly speaking, be known by definition the ‚area‘ of its changeability; this ‚area‘ cannot be anything changeable, otherwise a solid basis for the analysis would be missing; this ‚area‘ of values is then a certain actually infinite set. And hence, any potential infinity presupposes an actual infinity to be strictly usable in mathematics.“²¹ This is almost like a quotation from Gutberlet. Both Gutberlet and Cantor were convinced that infinity must be of divine origin, thus they would accept the statement that the existence of infinity, if only in our minds, can be considered a proof of God. Therefore, mathematics is much closer to theology than is usually admitted, hence, in their minds, mathematicians are theologians of sorts, who are interested in one of the primary attributes of God, namely his infinity, although they investigate this attribute with different means than the theologians.

²⁰ Adam Drozdek, *Beyond infinity: Augustine and Cantor*, in: *Laval théologique et philosophique* 51 (1995) 127–140.

²¹ In Herbert Meschkowski, *Probleme des Unendlichen: Werk und Leben Georg Cantors* (Braunschweig 1967) 250. Cantor corresponded with Gutberlet about this issue and even paid him a visit in Fulda, K. Gutberlet, *Eine Selbstbiographie*, 178.